

# Capacitive Page Count Sensor

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The sensor bases on the measurement of the voltage across the capacitor consisting of two metal plates with regular paper in between. The sensor uses a RC (Resistor-Capacitor) circuit where a voltage in form of a sine wave is connected to. The physical effect we are using is described using two formulas:  
**(explanations of the symbols are given at the end of the document)**

$$Q = C * U$$

and

$$C = \epsilon_0 * \epsilon_r * \frac{A}{d}$$

For paper:  $\epsilon_r = 5..6$

The capacity  $C$  decreases if the distance between two plates  $d$  increases. However, the charge  $Q$  of a capacitor stays constant, which implies an increase in the voltage  $U$  across the capacitor plates.

**Conclusion 1:** *If the distance  $d$  decreases the voltage  $U$  across the capacitor increases.*

(in the next section the voltage across the capacitor  $U$  is named  $U_c$  )

The voltage  $U_c$  is given by the resistor of the capacitor. This resistor is  $Z$  . In AC

circuits  $Z$  is given by the formula:  $Z = \frac{1}{w * C}$

This resistor can also be written as  $Z = \frac{U_c}{I}$

The current  $I$  in this circuit can be described using the overall resistor of the circuit:

$R_0 = \frac{U_0}{I}$ , whereby  $R_0 = \sqrt{R^2 + \frac{1}{(w * C)^2}}$  (standard formula for the resistor of a RC

circuit).

Using the formula of  $R_0$  and  $Z$  the voltage  $U_c$  can be written as follows:

$$U_c = U_0 * \frac{1}{\sqrt{R^2 * w^2 * C^2 + 1}}$$

The distance  $d$  is contained in the term of  $C$  .

$$U_c \propto \frac{k_1}{\sqrt{\frac{1}{d^2} + k_2}} \quad (k_1, k_2 \text{ are constants})$$

**Conclusion 2:** The formula  $U_c = U_0 * \frac{1}{\sqrt{R^2 * w^2 * C^2 + 1}}$  describes the voltage  $U_c$  across the capacitor based on the distance  $d$  between the capacitor's plates.

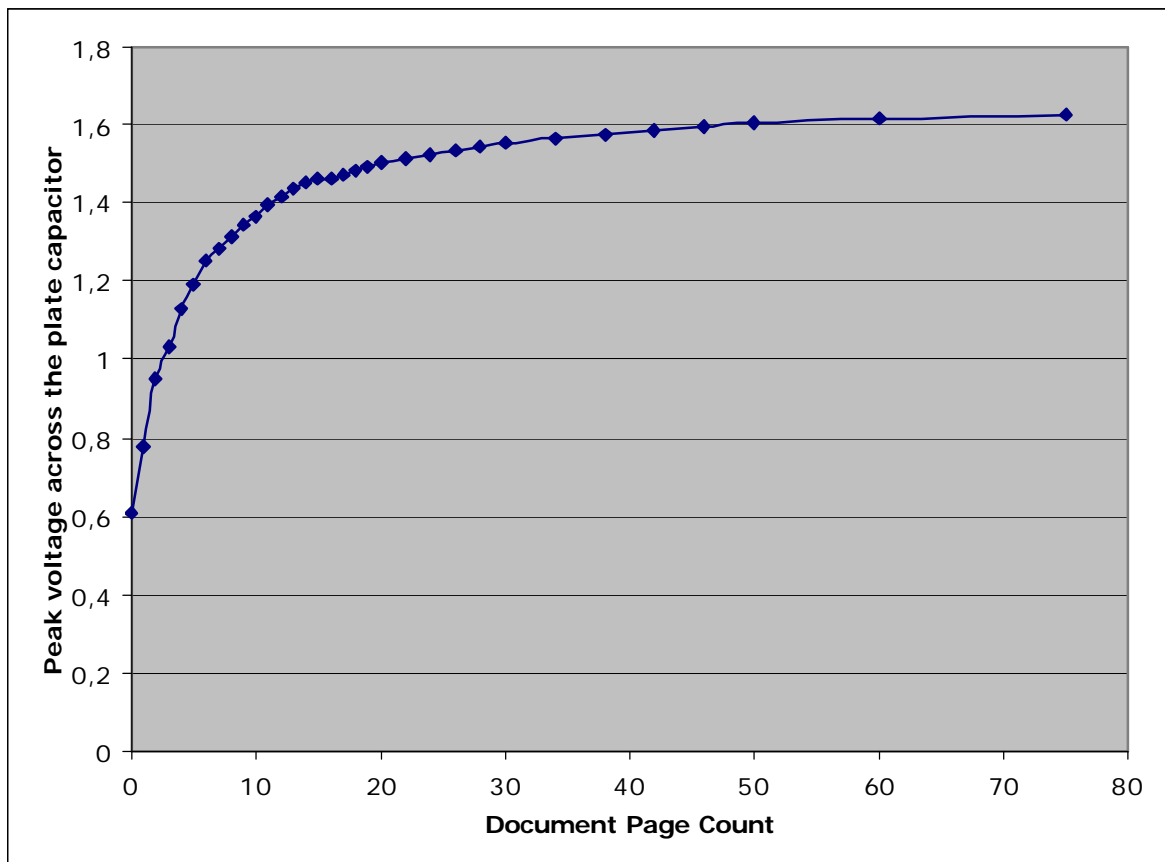
In our experiments we used

$$U_0 = 1.65 \text{ Volts}$$

$$R = 20 \text{ k}\Omega$$

$$f = 100 \text{ kHz}$$

Measuring the voltage  $U_c$  lead to the following correspondence:



The Document Page Count determines the distance  $d$  between the capacitor's plates.

**Note:** The formula given in Conclusion 2 leads to the same correspondence.

**Explanations:**

$U_c$  - voltage across the capacitor

$d$  - distance between the capacitor plates

$U_0$  - input voltage of the RC circuit (same as  $V_{IN}$  in the patent draft)

$R$  - resistor in the RC circuit

$Q$  - charge on the capacitor

$C$  - capacity of the capacitor

$\omega$  - angular frequency ( $\omega = 2 * \pi * f$ )

$f$  - Frequency of the input voltage  $U_0$

$Z$  - resistor of the capacitor

$R_0$  - overall resistor of the RC circuit

$I$  - current in the RC circuit

$\epsilon_0$  - dielectric coefficient

$\epsilon_r$  - relative dielectric coefficient, material specific ( for paper  $\epsilon_r \approx 5..6$ )

$A$  - surface area of the capacitor's plate